# El Filtro de Kalman

Diego Rodríguez-Losada

Robots Móviles.

UPM

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## Indice

- Probabilidad
- Derivación caso 1D
- KF lineal
- EKF
- Aplicaciones:
  - Estimación de un voltaje
  - Estimación de un misil
  - Localización 1D
  - Localización 2D

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# Probabilidad

| Event | Probability |
|---|---|
| A | $P(A) \in [0,1]$ |
| not A | $P(A') = 1 - P(A)$ |
| A or B | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ <br> $= P(A) + P(B)$ if A and B are mutually exclusive |
| A and B | $P(A \cap B) = P(A\|B)P(B)$ <br> $= P(A)P(B)$ if A and B are independent |
| A given B | $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ |

$$\mathrm{P}[a \le X \le b] = \int_a^b f(x)\,\mathrm{d}x.$$

$$F(x) = \int_{-\infty}^{x} f(u)\,\mathrm{d}u,$$

$$f(x) = \frac{\mathrm{d}}{\mathrm{d}x}F(x).$$



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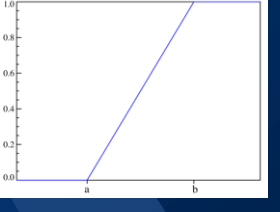
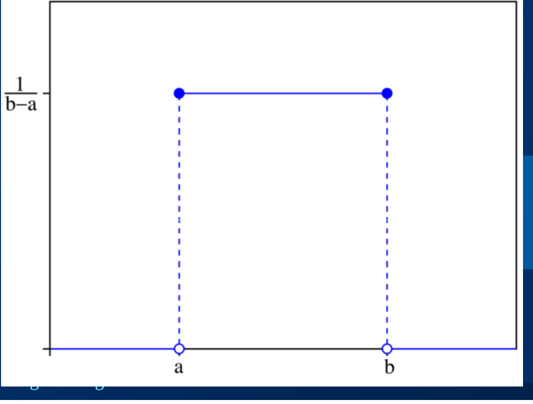
# Distribución uniforme

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b, \\ 0 & \text{for } x < a \text{ or } x > b \end{cases}$$



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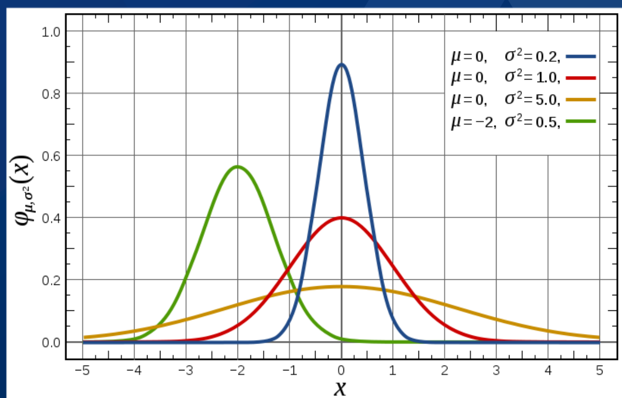
# Distribución normal

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



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# Propiedades Normal

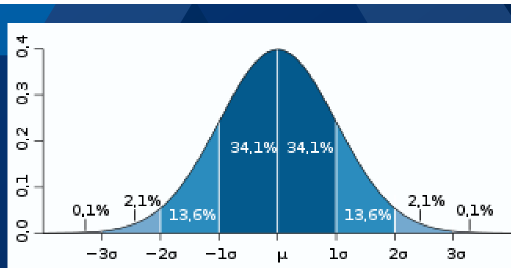
- Linealidad

$$x \sim N(m,\sigma^2) \Rightarrow ax \sim N(am, a^2\sigma^2).$$

$$x \sim N(m,\sigma^2) \Rightarrow x+u \sim N(m+u, \sigma^2).$$

$$x \sim N(m_x,\sigma_x^2), y \sim N(m_y,\sigma_y^2) \Rightarrow x+y \sim N(m_x+m_y, \sigma_x^2+\sigma_y^2).$$

- Intervalos



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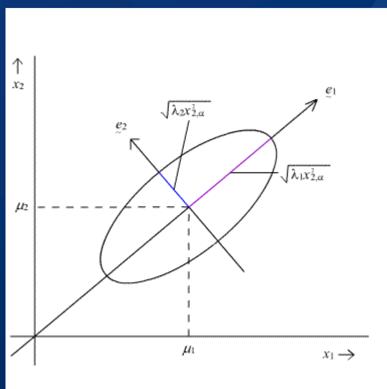
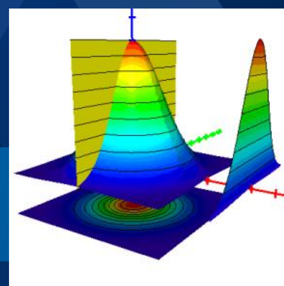
# Normal multivariable

$$f_X(x_1,\ldots,x_n) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\right)$$





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# Derivación 1D (var min)

$$p(x_i) = \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left[-\left(\frac{x_i - \bar{x}_i}{\sigma_i}\right)^2\right] \qquad (i = 1, 2).$$

$$\hat{x} = w x_1 + (1-w) x_2$$

$$\hat{\sigma}^2 = w^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 \ .$$

$$\frac{\partial}{\partial w} = 0$$

$$w_{opt} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$\begin{aligned} \hat{x} &= \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} x_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} x_2 \\ \hat{\sigma}^2 &= \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} \end{aligned}$$



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# Derivación 1D

- Reorganizando:

$$\begin{aligned} \hat{x}_2 &= \hat{x}_1 + \frac{\hat{\sigma}_1^2}{\hat{\sigma}_1^2 + \sigma_2^2} (x_2 - \hat{x}_1) \\ \hat{\sigma}_2^2 &= \left(1 - \frac{\hat{\sigma}_1^2}{\hat{\sigma}_1^2 + \sigma_2^2}\right) \hat{\sigma}_1^2 \ . \end{aligned}$$

- Podemos llamar: $K = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_1^2 + \sigma_2^2}$
- Entonces:

$$\begin{aligned} \hat{x}_2 &= \hat{x}_1 + K\,(x_2 - \hat{x}_1) \\ \hat{\sigma}_2^2 &= (1 - K)\,\hat{\sigma}_1^2 \ . \end{aligned}$$

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# Filtro Kalman Lineal (KF)

- Modelo del proceso

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{u}_k + \mathbf{w}_k$$

- Modelo de observación

$$\mathbf{z}_{k+1} = \mathbf{H}\mathbf{x}_{k+1} + \mathbf{v}_{k+1}$$

- Ruidos

$$\mathbf{w} \sim N(0, \mathbf{Q}), \quad \mathbf{v} \sim N(0, \mathbf{R})$$

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# Filtro Lineal: Predicción

$$\mathbf{x}_k \sim N(\hat{\mathbf{x}}_k, \mathbf{P}_k)$$

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{u}_k + \mathbf{w}_k$$

$$\bar{\mathbf{x}}_{k+1} = \mathbf{F}\hat{\mathbf{x}}_k + \mathbf{G}\mathbf{u}_k$$

$$\bar{\mathbf{P}}_{k+1} = \mathbf{F}\mathbf{P}_k\mathbf{F}^T + \mathbf{Q}$$



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# Filtro Lineal: Corrección

$$\bar{\mathbf{x}}_{k+1} \quad \bar{\mathbf{P}}_{k+1}$$

$$\mathbf{z}_{k+1} = \mathbf{H}\mathbf{x}_{k+1} + \mathbf{v}_{k+1}$$

$$\mathbf{S} = \mathbf{H}\bar{\mathbf{P}}_{k+1}\mathbf{H}^T + \mathbf{R}$$

$$\mathbf{K} = \bar{\mathbf{P}}_{k+1}\mathbf{H}^T\mathbf{S}^{-1}.$$

$$\hat{\mathbf{x}}_{k+1} = \bar{\mathbf{x}}_{k+1} + \mathbf{K}(\mathbf{z}_{k+1} - \mathbf{H}\bar{\mathbf{x}}_{k+1})$$

$$\mathbf{P}_{k+1} = \bar{\mathbf{P}}_{k+1} - \mathbf{K}\mathbf{S}\mathbf{K}^T,$$

$$\mathbf{x}_{k+1} \sim N(\hat{\mathbf{x}}_{k+1}, \mathbf{P}_{k+1})$$

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## Filtro Extendido (EKF)

- Modelo del proceso

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k+1))$$

- Modelo de observación

$$\mathbf{z}(k+1) = \mathbf{h}(\mathbf{x}(k+1)) + v(k+1)$$

- Ruidos

$$\mathbf{u}(k+1) \sim N\left(\hat{\mathbf{u}}(k+1), \mathbf{Q}(k+1)\right)$$
$$v(k+1) \sim N\left(0, \mathbf{R}(k+1)\right)$$

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# Predicción: Linealización modelo del proceso

$$
\begin{aligned}
\mathbf{x}(k+1) &= \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k+1)) \\
&\simeq \mathbf{f}(\hat{\mathbf{x}}(k|k), \hat{\mathbf{u}}(k+1)) + \\
&+ \mathbf{F}_x(k+1)\left(\mathbf{x}(k) - \hat{\mathbf{x}}(k|k)\right) + \\
&+ \mathbf{F}_u(k+1)\left(\mathbf{u}(k+1) - \hat{\mathbf{u}}(k+1)\right) + O^2
\end{aligned}
$$

$$
\mathbf{F}_x(k+1) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\substack{\hat{\mathbf{x}}(k|k) \\ \hat{\mathbf{u}}(k+1)}}
$$

$$
\mathbf{F}_u(k+1) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{\substack{\hat{\mathbf{x}}(k|k) \\ \hat{\mathbf{u}}(k+1)}}
$$

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17

# Filtro Extendido: Predicción

$$
\mathbf{x}(k) \sim N\left(\hat{\mathbf{x}}(k|k), \mathbf{P}(k|k)\right)
$$

$$
\mathbf{u}(k+1) \sim N\left(\hat{\mathbf{u}}(k+1), \mathbf{Q}(k+1)\right)
$$

$$
\begin{aligned}
\hat{\mathbf{x}}(k+1|k) &= \mathbf{f}(\hat{\mathbf{x}}(k|k), \hat{\mathbf{u}}(k+1)) \\
\mathbf{P}(k+1|k) &= \mathbf{F}_x(k+1)\mathbf{P}(k|k)\mathbf{F}_x^{\mathrm{T}}(k+1) \\
&+ \mathbf{F}_u(k+1)\mathbf{Q}(k+1)\mathbf{F}_u^{\mathrm{T}}(k+1)
\end{aligned}
$$

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18

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# Corrección: Linealización modelo de observación

$$\mathbf{z}(k+1) = \mathbf{h}(\mathbf{x}(k+1)) + v(k+1)$$
$$\simeq \mathbf{h}(\hat{\mathbf{x}}(k+1|k)) +$$
$$+\mathbf{H}_x(k+1)\left(\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)\right) + O^2$$

$$\mathbf{H}_x(k+1) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\substack{\hat{\mathbf{x}}\\ \mathbf{z}}}$$



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# Filtro Extendido: Corrección

$$\hat{\mathbf{x}}(k+1|k), \mathbf{P}(k+1|k)$$

Ganancia del filtro

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}_x^T(k+1)\mathbf{S}^{-1}(k+1)$$

Correccion del estado

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}(k+1)\hat{\mathbf{h}}(k+1)$$
$$\mathbf{P}(k+1|k+1) = \left[\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}_x(k+1)\right]\mathbf{P}(k+1|k)$$
$$= \mathbf{P}(k+1|k) - \mathbf{K}(k+1)\mathbf{S}(k+1)\mathbf{K}^T(k+1)$$

$$\hat{\mathbf{x}}(k+1|k+1), \mathbf{P}(k+1|k+1)$$

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# Corrección (Joseph)

$$\mathbf{P}(k+1|k+1) = \left(\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}\right)\mathbf{P}(k+1|k)\left(\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}\right)^T + \mathbf{K}(k+1)\mathbf{R}\mathbf{K}(k+1)^T$$

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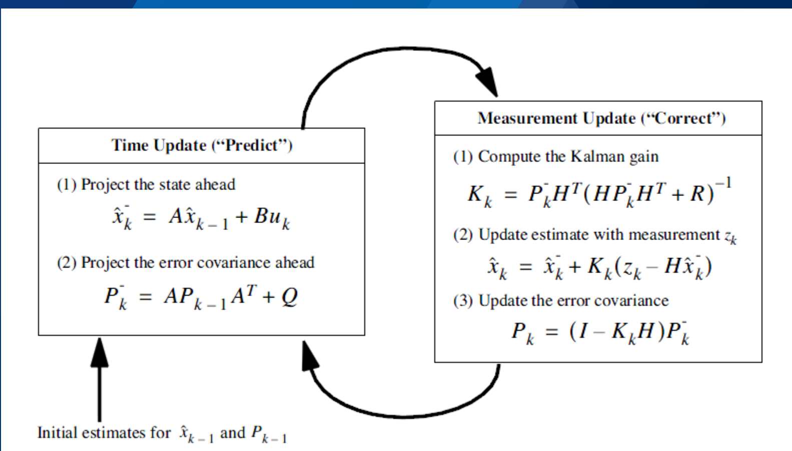
# Ciclo

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# Indice





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# Estimación de un voltaje

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# Estado y modelo del proceso

- Estado

$$\mathbf{x} = (V)$$

- Modelo del proceso

$$V(t+1) = V(t)$$

- Estimación inicial

$$\hat{\mathbf{x}}(0) = (4.5), P(0) = [0.5]$$



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# Observación y corrección

```
S=P+cov_obs;
K=P*inv(S);
inn=z-x;
x=x+K*inn;
P=P-K*S*K';
```

$$\mathbf{z} = V + \nu$$

$$\nu \sim N(0, \mathbf{R})$$

$$\mathbf{S} = \mathbf{H}\bar{\mathbf{P}}_{k+1}\mathbf{H}^T + \mathbf{R}$$

$$\mathbf{K} = \bar{\mathbf{P}}_{k+1}\mathbf{H}^T\mathbf{S}^{-1}.$$

$$\hat{\mathbf{x}}_{k+1} = \bar{\mathbf{x}}_{k+1} + \mathbf{K}(\mathbf{z}_{k+1} - \mathbf{H}\bar{\mathbf{x}}_{k+1})$$

$$\mathbf{P}_{k+1} = \bar{\mathbf{P}}_{k+1} - \mathbf{K}\mathbf{S}\mathbf{K}^T,$$

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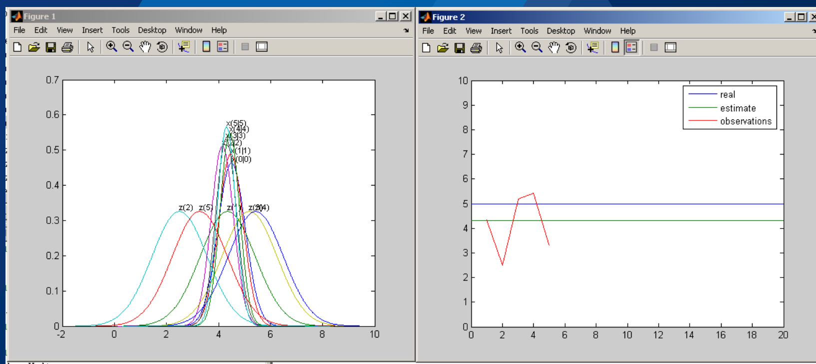
# Demo



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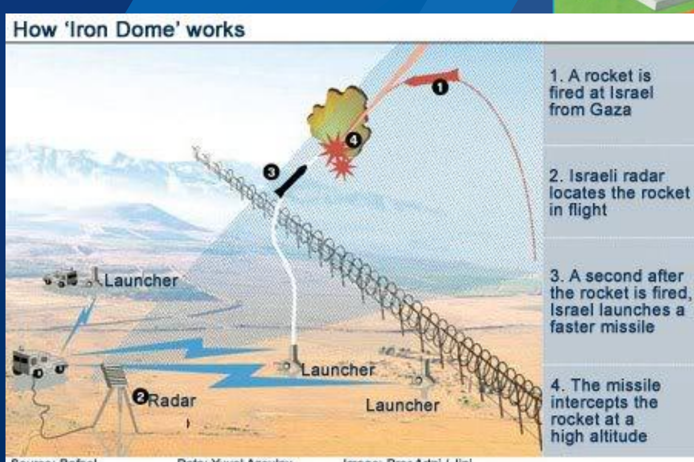
# Trayectoria proyectil



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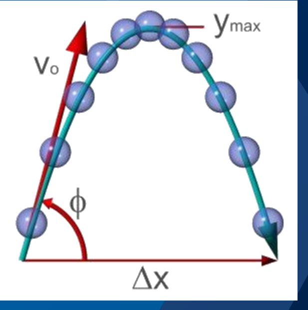
# Trayectoria proyectil

- Estado

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}$$



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# Modelo del proceso

```
function x=ProcessModel(x,t)
    x(1)=x(1)+x(3)*t;
    x(2)=x(2)+x(4)*t-4.9*t*t;
    x(3)=x(3);
    x(4)=x(4)-9.8*t;
end
```

$$x = x + v_x \Delta t$$

$$y = y + v_y \Delta t - \frac{1}{2} g \Delta t^2$$

$$v_x = v_x$$

$$v_y = v_y - g\Delta t$$

$$\mathbf{x} = f(\mathbf{x}) = \mathbf{F}\mathbf{x} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}$$

# Observación

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```
z= [ x_r(1)+sigma_obs*randn();
      x_r(2)+sigma_obs*randn()];
 %Jacobian
           H=[1 0 0 0;
              0 1 0 0];
```

$$\mathbf{H}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

$$\mathbf{z}=\mathrm{h}(\mathbf{x})+v=\begin{pmatrix}x\\ y\end{pmatrix}+v$$

$$v\sim N(0,\mathbf{R})$$

$$\text{ya que } \mathbf{x}=\begin{pmatrix}x\\ y\\ v_x\\ v_y\end{pmatrix}$$



# Corrección

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$$\hat{\mathbf{x}}_{k+1}=\bar{\mathbf{x}}_{k+1}+\mathbf{K}(\mathbf{z}_{k+1}-\mathbf{H}\bar{\mathbf{x}}_{k+1})$$
$$\mathbf{P}_{k+1}=\bar{\mathbf{P}}_{k+1}-\mathbf{K}\mathbf{S}\mathbf{K}^T,$$

$$\mathbf{S}=\mathbf{H}\bar{\mathbf{P}}_{k+1}\mathbf{H}^T+\mathbf{R}$$
$$\mathbf{K}=\bar{\mathbf{P}}_{k+1}\mathbf{H}^T\mathbf{S}^{-1}.$$

```
%Next kalman updated position (k|k)
        S=H*P*H'+R;
        K=P*H'*inv(S);
        inn=z-x_e(1:2);
        x_e=x_e+K*inn;
        P=P-K*S*K';
```

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# Inicialización

$$\hat{\mathbf{x}}(0)=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}, P(0)=\begin{bmatrix}\infty & 0 & 0 & 0\\ 0 & \infty & 0 & 0\\ 0 & 0 & \infty & 0\\ 0 & 0 & 0 & \infty\end{bmatrix}$$

$$x(1|1)=\mathbf{z}_x(1)$$
$$y(1|1)=\mathbf{z}_y(1)$$
$$x(2|2)=\mathbf{z}_x(2)$$
$$y(2|2)=\mathbf{z}_y(2)$$
$$v_y(2|2)=\frac{x(2|2)-x(1|1)}{\Delta t}$$
$$v_y(2|2)=\frac{y(2|2)-y(1|1)}{\Delta t}$$



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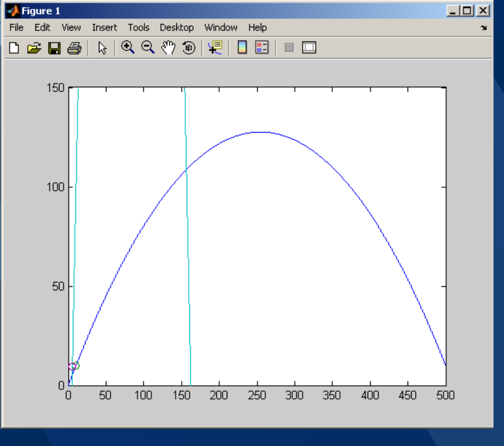
# Demo



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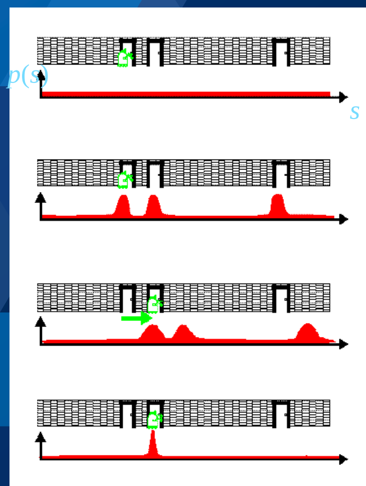
# Localización 1D



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# Predicción

$x_k \sim N(\hat{x}_k, \sigma_k^2)$

$x_{k+1} = x_k + u_k + w_k$

$x_k + u_k \sim N(\hat{x}_k + u_k, \sigma_k^2).$

$w_k \sim N(0, \sigma_u^2)$

$x_{k+1} = (x_k + u_k) + w_k \sim N(\hat{x}_k + u_k, \sigma_k^2 + \sigma_u^2).$

$x_{k+1} \sim N(\bar{x}_{k+1}, \bar{\sigma}_{k+1}^2),$

$$\begin{aligned} \bar{x}_{k+1} &= \hat{x}_k + u_k, \\ \bar{\sigma}_{k+1}^2 &= \sigma_k^2 + \sigma_u^2. \end{aligned}$$

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## Observación

$$z_{k+1} = x_{k+1} + v_{k+1}$$

$$z_{k+1} = y - x_{k+1}$$

$y = 6$ posicion conocida baliza

$$-v_{k+1} \sim N(0, \sigma_z^2).$$

$$x_{k+1} = -v_{k+1} + z_{k+1}.$$

$$x_{k+1} \sim N(z_{k+1}, \sigma_z^2).$$

Modelo inverso de observación



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## Derivación: Información

$$I_{prior} = \frac{1}{\bar{\sigma}_{k+1}^2}, \qquad I_{obs} = \frac{1}{\sigma_z^2}.$$

$$I_{total} = I_{prior} + I_{obs} = \frac{1}{\bar{\sigma}_{k+1}^2} + \frac{1}{\sigma_z^2}.$$

$$\sigma_{k+1}^2 = \frac{1}{I_{total}} = \frac{1}{\frac{1}{\bar{\sigma}_{k+1}^2} + \frac{1}{\sigma_z^2}} = \frac{\bar{\sigma}_{k+1}^2 \sigma_z^2}{\bar{\sigma}_{k+1}^2 + \sigma_z^2}.$$

$$\hat{x}_{k+1} = \frac{I_{prior}}{I_{total}} \bar{x}_{k+1} + \frac{I_{obs}}{I_{total}} z_{k+1} = \frac{\sigma_z^2}{\bar{\sigma}_{k+1}^2 + \sigma_z^2} \bar{x}_{k+1} + \frac{\bar{\sigma}_{k+1}^2}{\bar{\sigma}_{k+1}^2 + \sigma_z^2} z_{k+1}.$$

# Derivación II

$$\mathbf{S} = \mathbf{H}\bar{\mathbf{P}}_{k+1}\mathbf{H}^T + \mathbf{R}$$
$$\mathbf{K} = \bar{\mathbf{P}}_{k+1}\mathbf{H}^T\mathbf{S}^{-1}.$$

$$\hat{\mathbf{x}}_{k+1} = \bar{\mathbf{x}}_{k+1} + \mathbf{K}(\mathbf{z}_{k+1} - \mathbf{H}\bar{\mathbf{x}}_{k+1})$$
$$\mathbf{P}_{k+1} = \bar{\mathbf{P}}_{k+1} - \mathbf{K}\mathbf{S}\mathbf{K}^T,$$

$$\begin{aligned}
\hat{x}_{k+1} &= \bar{x}_{k+1} + \frac{\bar{\sigma}^2_{k+1}}{\bar{\sigma}^2_{k+1}+\sigma^2_z}(z_{k+1} - \bar{x}_{k+1}) \\
\sigma^2_{k+1} &= \bar{\sigma}^2_{k+1}(1 - \frac{\bar{\sigma}^2_{k+1}}{\bar{\sigma}^2_{k+1}+\sigma^2_z}) \\
&= \bar{\sigma}^2_{k+1} - \bar{\sigma}^2_{k+1}\frac{\bar{\sigma}^2_{k+1}}{\bar{\sigma}^2_{k+1}+\sigma^2_z} \\
&= \bar{\sigma}^2_{k+1} - \frac{\bar{\sigma}^2_{k+1}}{\bar{\sigma}^2_{k+1}+\sigma^2_z}(\bar{\sigma}^2_{k+1} + \sigma^2_z)\frac{\bar{\sigma}^2_{k+1}}{\bar{\sigma}^2_{k+1}+\sigma^2_z}.
\end{aligned}$$

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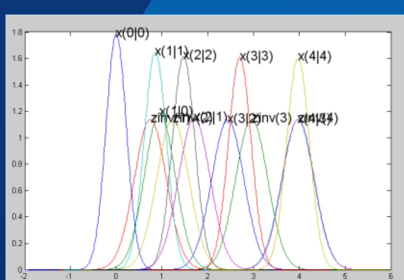
39

# Demo




- Robot se mueve 1 m
- sigma_u=0.25
- sigma_z=0.35
- Una baliza a 6 metros

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40

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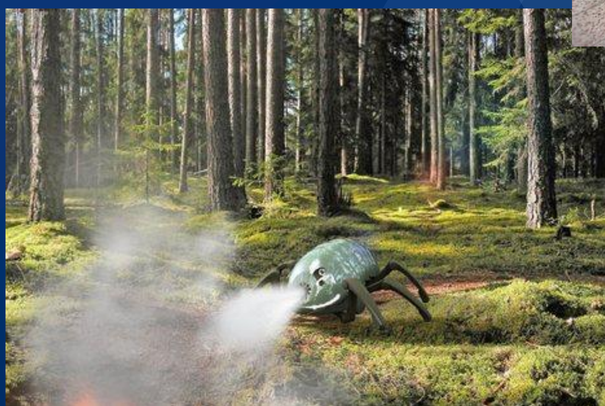
## Localización robot



41

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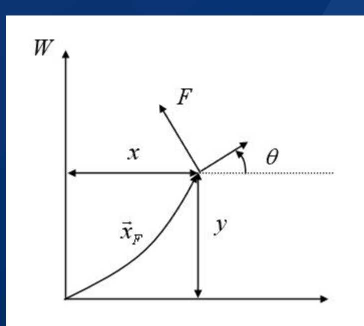
## Localización

$$M = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & & y_n \end{pmatrix}$$




$$\mathbf{x}_R = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$

$$\mathbf{x}_R \sim \mathrm{N}(\hat{\mathbf{x}}_R(k \mid k), \mathbf{P}(k \mid k))$$

42

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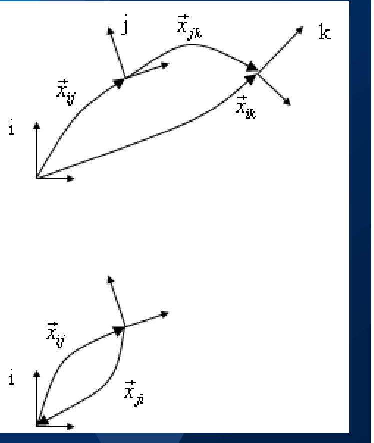
# Transformaciones relativas

- Composición e inversión

$$\mathbf{x}_{ik} = \mathbf{x}_{ij} \oplus \mathbf{x}_{jk} \Leftrightarrow \begin{cases} x_{ik} = x_{ij} + x_{jk}\cos\theta_{ij} - y_{jk}\sin\theta_{ij} \\ y_{ik} = y_{ij} + x_{jk}\sin\theta_{ij} + y_{jk}\cos\theta_{ij} \\ \theta_{ik} = \theta_{ij} + \theta_{jk} \end{cases}$$

$$\mathbf{x}_{ji} = \ominus\mathbf{x}_{ij} \Leftrightarrow \begin{cases} x_{ji} = -x_{ij}\cos\theta_{ij} - y_{ij}\sin\theta_{ij} \\ y_{ji} = x_{ij}\sin\theta_{ij} - y_{ij}\cos\theta_{ij} \\ \theta_{ji} = -\theta_{ij} \end{cases}$$



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# Jacobianas

$$\mathbf{x}_c = \mathbf{x}_a \oplus \mathbf{x}_b \Leftrightarrow \begin{cases} x_c = x_a + x_b\cos\theta_a - y_b\sin\theta_a \\ y_c = y_a + x_b\sin\theta_a + y_b\cos\theta_a \\ \theta_c = \theta_a + \theta_b \end{cases}$$

$$\mathbf{x}_c = \mathbf{x}_a \oplus \mathbf{x}_b \Leftrightarrow \begin{cases} x_c = x_a + x_b\cos\theta_a - y_b\sin\theta_a \\ y_c = y_a + x_b\sin\theta_a + y_b\cos\theta_a \end{cases}$$

$$\mathbf{J}_1(\mathbf{x}_c) = \mathbf{J}_1(\hat{\mathbf{x}}_a, \hat{\mathbf{x}}_b) = \left.\frac{\partial \mathbf{x}_c}{\partial \mathbf{x}_a}\right|_{\hat{\mathbf{x}}_a, \hat{\mathbf{x}}_b}$$

$$\mathbf{J}_2(\mathbf{x}_c) = \mathbf{J}_2(\hat{\mathbf{x}}_a, \hat{\mathbf{x}}_b) = \left.\frac{\partial \mathbf{x}_c}{\partial \mathbf{x}_b}\right|_{\hat{\mathbf{x}}_a, \hat{\mathbf{x}}_b}$$

# Movimiento del robot I

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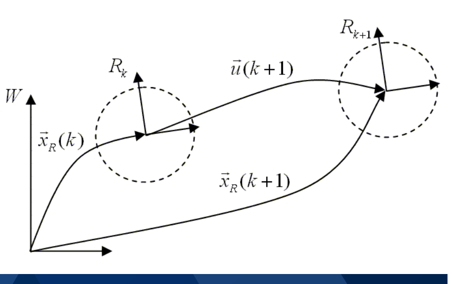
- Medida odométrica

$$\mathbf{u}(k+1) \sim \mathrm{N}(\hat{\mathbf{u}}(k+1), \mathbf{Q}(k+1))$$




- Ecuación de predicción

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k+1)) = \mathbf{x}(k) \oplus \mathbf{u}(k+1)$$



# Movimiento del robot II

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- Predicción del estado en k+1|k

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k+1)) = \mathbf{x}(k) \oplus \mathbf{u}(k+1)$$

coste computacional O(1)

$$\hat{\mathbf{x}}_R(k+1|k) = \hat{\mathbf{x}}_R(k|k) \oplus \hat{\mathbf{u}}(k+1)$$

$\hat{\mathbf{x}}(k|k)$

$\mathbf{P}(k|k)$

$\hat{\mathbf{u}}(k+1), \mathbf{Q}(k+1)$

$$\mathbf{F}_x(k+1) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\substack{\hat{\mathbf{x}}_R(k|k) \\ \hat{\mathbf{u}}(k+1)}} = \mathbf{J}_1(\hat{\mathbf{x}}_R(k|k), \hat{\mathbf{u}}(k+1))$$

$$\mathbf{F}_u(k+1) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{\substack{\hat{\mathbf{x}}_R(k|k) \\ \hat{\mathbf{u}}(k+1)}} = \mathbf{J}_2(\hat{\mathbf{x}}_R(k|k), \hat{\mathbf{u}}(k+1))$$

$$\mathbf{P}(k+1|k) = \mathbf{F}_x(k+1)\mathbf{P}(k|k)\mathbf{F}_x^{\mathrm{T}}(k+1) + \mathbf{F}_u(k+1)\mathbf{Q}(k+1)\mathbf{F}_u^{\mathrm{T}}(k+1)$$

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# Observaciones

- En el instante k+1 el robot realiza la observación de 'm' objetos.
  - Asociacion de datos conocida
  - Asociación de datos desconocida:
    - Decidir a cual objeto corresponde cada observación, o si es un objeto nuevo.
      - Test de compatibilidad individual
      - Test de compatibilidad conjunta

$$\mathbf{z} = \begin{pmatrix} z_{x_1} & z_{x_2} & \cdots & z_{x_m} \\ z_{y_1} & z_{y_2} & \cdots & z_{y_m} \end{pmatrix}$$

$$i = \begin{pmatrix} i_1 & i_2 & \cdots & i_m \end{pmatrix}$$



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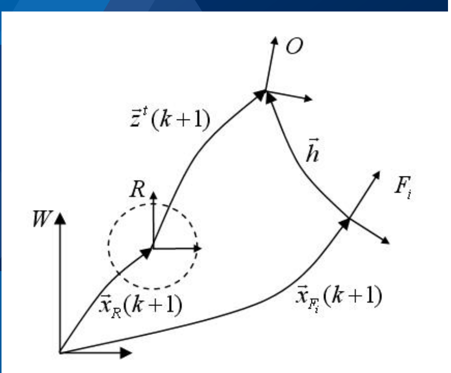
# Corrección I

- Ecuación implícita de medida

$$\mathbf{h}(\mathbf{z}^t(k+1), \mathbf{x}_R(k+1)) = 0$$




$$\mathbf{h}(\mathbf{z}^t(k+1), \mathbf{x}_R(k+1)) = \ominus \mathbf{x}_{F_i}(k+1) \oplus \mathbf{x}_R(k+1) \oplus \mathbf{z}^t(k+1) = 0$$

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## For point landmarks

index=indices(i)

$$\mathbf{h}(k+1)=\mathbf{h}(\mathbf{z}_i(k+1),\mathbf{x}_R(k+1),\mathbf{M}) =$$

$$= -\mathbf{M}_{\text{index}} + \mathbf{x}_R(k+1)\oplus \mathbf{z}_i(k+1)=0$$

49

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## Corrección II

$$\mathbf{h}(\mathbf{z}^t(k+1),\mathbf{x}_R(k+1)) = \ominus\mathbf{x}_{F_i}(k+1)\oplus\mathbf{x}_R(k+1)\oplus\mathbf{z}^t(k+1)=0$$

$$-\hat{\mathbf{h}}(k+1) = -\hat{\mathbf{h}}(\mathbf{z}(k+1),\hat{\mathbf{x}}_R(k+1|k))\neq 0$$

$$\mathbf{z}^t(k+1)\sim N\left(\mathbf{z}(k+1),\mathbf{R}(k+1)\right)$$

$$\hat{\mathbf{x}}_R(k+1|k)$$

$$\mathbf{P}(k+1|k)$$

$$\mathbf{H}_x(k+1)=\left.\frac{\partial\mathbf{h}}{\partial\mathbf{x}_R}\right|_{\substack{\hat{\mathbf{x}}_R\\ \mathbf{z}_i}} = \mathbf{J}_1(\hat{\mathbf{x}}_R,\mathbf{z}_i) \qquad 2\text{x}3$$

$$\mathbf{H}_z(k+1)=\left.\frac{\partial\mathbf{h}}{\partial\mathbf{z}_i}\right|_{\substack{\hat{\mathbf{x}}_R\\ \mathbf{z}_i}} = \mathbf{J}_2(\hat{\mathbf{x}}_R,\mathbf{z}_i) \qquad 2\text{x}2$$

$$\mathrm{S}(k+1) = \mathrm{H}_x(k+1)\mathrm{P}(k+1|k)\mathrm{H}_x^{\mathrm{T}}(k+1)+\mathrm{H}_z(k+1)\mathrm{R}(k+1)\mathrm{H}_z^{\mathrm{T}}(k+1)$$

50

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# Corrección EKF

$\hat{\mathbf{x}}_R(k+1|k)$
$\mathbf{P}(k+1|k)$

$\mathbf{S}(k+1)$

$\hat{\mathbf{h}}(k+1)$

$$\mathbf{H}_x(k+1) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_R}\right|_{\substack{\hat{\mathbf{x}}_R \\ \mathbf{z}_i}} = \mathbf{J}_1(\hat{\mathbf{x}}_R, \mathbf{z}_i)$$

$$\mathbf{H}_z(k+1) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{z}_i}\right|_{\substack{\hat{\mathbf{x}}_R \\ \mathbf{z}_i}} = \mathbf{J}_2(\hat{\mathbf{x}}_R, \mathbf{z}_i)$$

Ganancia del filtro

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}_x^T(k+1)\mathbf{S}^{-1}(k+1)$$

Correccion del estado

$$\hat{\mathbf{x}}_R(k+1|k+1) = \hat{\mathbf{x}}_R(k+1|k) - \mathbf{K}(k+1)\hat{\mathbf{h}}(k+1)$$

$$\mathbf{P}(k+1|k+1) = \left[\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}_x(k+1)\right]\mathbf{P}(k+1|k)$$

$\hat{\hat{x}}(k+1|k+1)$
$\mathrm{P}(k+1|k+1)$

51

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# Asociación datos desconocida

*foreach* $\mathbf{z}_i \in \mathbf{z}$
    *foreach* $\mathbf{x}_{F_j} \in M$
        d=dist($-\hat{\mathbf{h}}(k+1), 0$);
        if(d<min)
            $d \leftarrow \min$
        *end*
    *end*
*end*

52

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# Test de Mahalanobis individual

$\hat{\mathbf{h}}(k+1)$ S(k + 1)

$$\hat{\mathbf{h}}^T(k+1)\mathbf{S}^{-1}(k+1)\hat{\mathbf{h}}^T(k+1) < \chi^2_{\dim(\mathbf{h}(k+1)),\alpha}$$

Emparejamiento no válido — NO

SI — Emparejamiento válido

**Estrategias:**

**-Nearest Neighbour NN: vecino más cercano**

**-Todos los emparejamientos válidos.**

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53

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# Test Compatibilidad Conjunta

- Robustez en la asociación.

$\hat{\mathbf{h}}_{acum}(k+1)$ $S_{acum}(k+1)$

$$\hat{\mathbf{h}}^{T}_{acum}{}^{T}\mathbf{S}^{-1}_{acum}\hat{\mathbf{h}}_{acum} < \chi^2_{\dim(\mathbf{h}_{acum}(k+1)),\alpha}$$

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54

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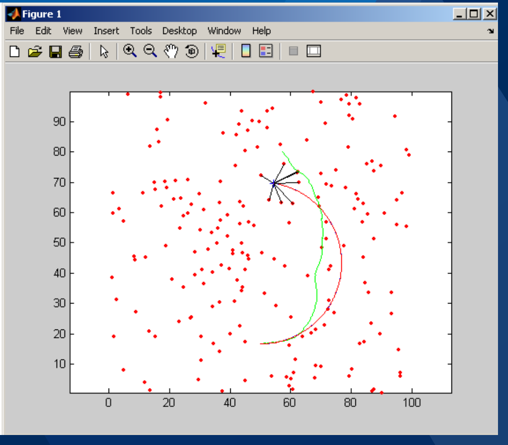
# Demo



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# Derivación bayesiana

$$p(\mathbf{x}_t \mid \mathbf{z}^{t-1},\mathbf{u}^t) \sim \exp\left(-\frac{(\mathbf{x}_t-\hat{\mathbf{x}}_t)^T\,\mathbf{P}_t^{-1}(\mathbf{x}_t-\hat{\mathbf{x}}_t)}{2}\right) \quad \text{Estimation}$$

$$p(\mathbf{z}_t \mid \mathbf{x}_t) \sim \exp\left(-\frac{(\mathbf{z}_t-\mathbf{h}(\mathbf{x}_t))^T\,\mathbf{R}_t^{-1}(\mathbf{z}_t-\mathbf{h}(\mathbf{x}_t))}{2}\right) \quad \text{Measurement model}$$

$$p(\mathbf{x}_t \mid \mathbf{z}^t,\mathbf{u}^t) \overset{Bayes}{\sim} p(\mathbf{z}_t \mid \mathbf{x}_t,\mathbf{z}^{t-1},\mathbf{u}^t)\,p(\mathbf{x}_t \mid \mathbf{z}^{t-1},\mathbf{u}^t) =$$

$$\overset{Markov}{\sim} p(\mathbf{z}_t \mid \mathbf{x}_t)\,p(\mathbf{x}_t \mid \mathbf{z}^{t-1},\mathbf{u}^t) =$$

$$\overset{Gauss}{\sim} \exp\left(-\frac{(\mathbf{x}_t-\hat{\mathbf{x}}_t)^T\,\mathbf{P}_t^{-1}(\mathbf{x}_t-\hat{\mathbf{x}}_t)}{2}-\frac{(\mathbf{z}_t-\mathbf{h}(\mathbf{x}_t))^T\,\mathbf{R}_t^{-1}(\mathbf{z}_t-\mathbf{h}(\mathbf{x}_t))}{2}\right)$$

# Derivación bayesiana

$$\boldsymbol{\mu}_t = \arg\max_{\mathbf{x}_t}(p(\mathbf{x}_t \mid \mathbf{z}^t, \mathbf{u}^t))$$

$$\mathbf{h}(\mathbf{x}_t) = \mathbf{h}(\hat{\mathbf{x}}_t) + \mathbf{H}\left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right) + O\left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right)^2 \approx \mathbf{h}(\hat{\mathbf{x}}_t) + \mathbf{H}\mathbf{x}_t - \mathbf{H}\hat{\mathbf{x}}_t$$

$$\mathbf{f} = \left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right)^T \mathbf{P}_t^{-1}\left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right) + \left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t) - \mathbf{H}\mathbf{x}_t + \mathbf{H}\hat{\mathbf{x}}_t\right)^T \mathbf{R}_t^{-1}\left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t) - \mathbf{H}\mathbf{x}_t + \mathbf{H}\hat{\mathbf{x}}_t\right)$$

$$\frac{\delta \mathbf{f}}{\delta \mathbf{x}_t} = 2\mathbf{P}_t^{-1}\left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right) - 2\mathbf{H}^T\mathbf{R}_t^{-1}\left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t) - \mathbf{H}\mathbf{x}_t + \mathbf{H}\hat{\mathbf{x}}_t\right) = 0$$

$$\mathbf{P}_t^{-1}\mathbf{x}_t - \mathbf{P}_t^{-1}\hat{\mathbf{x}}_t - \mathbf{H}^T\mathbf{R}_t^{-1}\left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t)\right) + \mathbf{H}^T\mathbf{R}_t^{-1}\mathbf{H}\vec{x}_t - \mathbf{H}^T\mathbf{R}_t^{-1}\mathbf{H}\hat{\mathbf{x}}_t = 0$$

$$\left(\mathbf{P}_t^{-1} + \mathbf{H}^T\mathbf{R}_t^{-1}\mathbf{H}\right)\mathbf{x}_t = \left(\mathbf{P}_t^{-1} + \mathbf{H}^T\mathbf{R}_t^{-1}\mathbf{H}\right)\hat{\mathbf{x}}_t + \mathbf{H}^T\mathbf{R}_t^{-1}\left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t)\right)$$

$$\boldsymbol{\mu}_t = \hat{\mathbf{x}}_t + \left(\mathbf{P}_t^{-1} + \mathbf{H}^T\mathbf{R}_t^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{R}_t^{-1}\left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t)\right)$$

$$\boldsymbol{\mu}_t = \hat{\mathbf{x}}_t + \mathbf{P}_t\mathbf{H}^T\left(\mathbf{H}\mathbf{P}_t\mathbf{H}^T + \mathbf{R}_t\right)^{-1}\left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t)\right) = \hat{\mathbf{x}}_t + \mathbf{K}\left(\mathbf{z}_t - \mathbf{h}(\hat{\mathbf{x}}_t)\right)$$



# Derivación bayesiana

$$P' = E\left(\left(\vec{x}_t - \vec{\mu}'_t\right)\left(\vec{x}_t - \vec{\mu}'_t\right)^T\right) = E\left(\left(\vec{x}_t - \vec{\mu}_t - W\gamma\right)\left(\vec{x}_t - \vec{\mu}_t - W\gamma\right)^T\right) =$$

$$E\left(\left(\vec{x}_t - \vec{\mu}_t\right)\left(\vec{x}_t - \vec{\mu}_t\right)^T - W\gamma\left(\vec{x}_t - \vec{\mu}_t\right)^T - \left(\vec{x}_t - \vec{\mu}_t\right)(W\gamma)^T + (W\gamma)(W\gamma)^T\right) =$$

$$E\left(\left(\vec{x}_t - \vec{\mu}_t\right)\left(\vec{x}_t - \vec{\mu}_t\right)^T\right) - E\left(W\gamma\left(\vec{x}_t - \vec{\mu}_t\right)^T\right) - E\left(\left(\vec{x}_t - \vec{\mu}_t\right)(W\gamma)^T\right) + E\left(W\gamma\gamma^T W^T\right) =$$

$$P' = P + WSW^T$$